MTH603 - Numerical Analysis
FINAL TERM 2013 Papers

Q. from the following table of values, construct backward difference table.

x	5	10	15	20
$f(x)$	1.6094	2.3026	2.7081	2.9957

X	F(x)			
5	1.6094			
10	2.3026	0.6932		
15	2.7081	0.4055	-0.2877	
20	2.9957	0.2876	-0.1179	-0.1698

Q. Use Euler's method to approximate y when x=1, given that

$$\frac{dy}{dx} = \frac{y-2x}{y+2x}, y(0) = 1, taking\ h = 1$$

Solution:

h = 1 , x = 1 and , $\frac{dy}{dx} = \frac{y-2x}{y+2x}$

Thus $y_1 = y + hf(x, y)$ where y_0=1 and x_0=1

$$y_1 = y + h\frac{y-2x}{y+2x}$$

$$y_1 = 1 + 1\frac{1-2(1)}{1+2(1)}$$

$$y_1 = -1.0$$

Q. Evaluate the integral

$$\int_{\frac{\pi}{2}}^{\pi} \sin x dx$$

Using Simpson's 3/8 rule

Take h= $\frac{\pi}{4}$

Solution:

X	$\frac{\pi}{4}$	$\frac{\pi}{2}$	$\frac{3\pi}{4}$	π
Y= $\sin x dx$	0.8509	0.8940	0.0884	- 0.8012

Using Simpson's 3/8 rule

$$\int_{\frac{\pi}{2}}^{\pi} \sin x dx = \frac{3}{8} h(y_0 + 3y_1 + 3y_2 + y_3)$$

$$= \frac{3}{8} \times \frac{\pi}{4} (0.8509 + 3(0.8940) + 3(0.0884) - 0.8012)$$

=50.5727

Q. Use Runge-Kutta Method of order four to find the values of $k_1, k_2, k_3 \; and \; k_4$ for the initial value problem

$$y^{/} = x + y, y(0) = 1, h = 0.1$$

Solution:

We know that

$y^{/} = x + y, y(0) = 1, h = 0.1$ X=0 and Y=1

In this problem,

$$f(x, y) = x + y, \qquad h = 0.1, x_0 = 0, y_0 = 1$$

$$k_1 = hf(x_0, y_0) = 0.1(1) = 0.1$$

$$k_2 = hf\left[x_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right]$$

$$k_2 = 0.1[0 + 0.05, 1 + 0.05]$$

$$k_2 = 0.1100$$

$$k_3 = hf(x_0 + 0.05, y_0 + 0.05)$$

$$k_3 = 0.1(0.05 + 1.055)$$

$$k_3 = 0.1105$$

$$k_4 = 0.1(0.1 + 1.1105)$$

$$k_4 = 0.12105$$

Q. Obtain numerically the solution of

$$y^{/} = x^2 + y^2, \;\; y(0) = 1$$

Using Euler's method to find y at x =2, h=2

Solution:

Since h=2, so we calculate value at x=2 y(0)=1, here X_0=2, Y_0=1

$$y_1 = y_0 + hf(x_0, y_0)$$
$$= y_0 + h(x^2, y^2)$$
$$= 1 + 2(2^2 + 1^2)$$
$$= 11$$

Q. Use Runge-Kutta Method of order four to find the values of k_1 and k_2 for the initial value problem

$$y' = 1 + xy, y(0) = 2, h = 0.2 \qquad (02)$$

Solution:

$$k_1 = hf(t_0, y_0) = 1.(2)(2) = 4$$
$$k_2 = hf\left[t_0 + \frac{h}{2}, y_0 + \frac{k_1}{2}\right]$$
$$k_2 = 0.2\left[1 + 0.1, 2 + \frac{4}{2}\right]$$
$$k_2 = 1.0200$$

Q. Evaluate the integral

$$\int_0^{\frac{\pi}{2}} (\cos x + 2)dx$$

Using Simpson's 1/3 rule

Take h= $\frac{\pi}{4}$

Solution:

X	**0**	$\frac{\pi}{4}$	$\frac{\pi}{2}$
Y = cosxdx	**1**	**0.5253**	**- 0.4481**

Simpson 1 / 3 rule

$$\int_{\frac{\pi}{2}}^{\pi} \cos xdx = \frac{3}{h}(y_0 + y_2 + 4y_1 + 2)$$

$$= \frac{3}{45}(1 - 0.4481 + 4(0.5253) + 2)$$

$$= 0.3102$$

Q. Using Adam-Moulton Predictor-Corrector Formula find the predicted value of f(0.4) from Ordinary Differential Equation

$$y' = 1 + xy \quad ; \quad y(0) = 0 \;; \quad h = 0.1$$

with the help of following table.

X	0	0.1	0.2	0.3
Y	0	0.1007	0.2056	0.3199

Solution:

Here,

$$f(x,y)=1+xy$$
$$y_0=1+x_0y_0=1+(0)(0)=1$$
$$y_1=1+x_1y_1=1+(0.1)(0.1007)=1.0101$$
$$y_2=1+x_2y_2=1+(0.2)(0.2056)=1.0411$$
$$y_3=1+x_3y_3=1+(0.3)(0.3199)=1.0960$$

Now, using Predictor Formula

$$y_4=y_3+\frac{h}{24}(55y'_3-59y'_2+37y'_1-9y'_0)$$

Put the values in it. And get the answer

Q. Consider the following table of values

x	y=F(x)	∇y	$\nabla^2 y$	$\nabla^3 y$	$\nabla^4 y$	$\nabla^5 y$
1	2					
1.1	2.1	0.1				
1.2	2.3	0.2	0.1			
1.3	2.7	0.4	0.2	0.1		
1.4	3.5	0.8	0.4	0.2	0.1	
1.5	4.5	1	0.2	-0.2	-0.4	-0.5

$p=\frac{x-x_n}{h}=\frac{1.45-1.5}{0.1}=-0.5$, ∇y_n = 1 , $\nabla^2 y_n$ = .2 , $\nabla^3 y_n$ = - .2 , $\nabla^4 y_n$ = -.4 , $\nabla^5 y_n$ = -.5

Use Newton's Backward Difference Formula to estimate the value of f(1.45) .

Solution:

$$y_x = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n$$
$$+\frac{p(p+1)(p+2)(p+3)}{4!}\nabla^4 y_0 + \frac{p(p+1)(p+2)(p+3)(p+4)}{5!}\nabla^5 y_0$$
$$y_x = 4.5 - 0.5 - 0.025 + 0.0125 + 0.015625 + 0.068359$$
$$= 4.0714$$

Q. Find $F(h)$, using Richardson's extrapolation limit, while finding $y^{/}(0.1)$ to the function $y = x^2$ with h =0.05. **3 Marks**

Solution:

$$x = 0.1362$$
$$F(h) = \frac{y(x+h) - y(x-h)}{2h}$$

Q.If $F(h) = 256.2354$ and $F(\frac{h}{2}) = 257.1379$, then find $F_1(\frac{h}{2})$ using Richardson's extrapolation limit. **(02)**

Solution:

$$F_1\left[\frac{h}{2}\right] = \frac{4F\left[\frac{h}{2}\right] - F\left[\frac{h}{2}\right]}{4-1}$$

Put these values in the formula and get the answer.

Q. If, in solving a given differential equation,
$y_0^{/} = 0.1, y_1^{/} = 1.1, y_2^{/} = 1.2, y_3^{/} = 1.3, h = 1, y_3 = 2$

Then find y_4 by Adam-Moultan's Predictor formula.

Solution:

$$y_4 = y_3 + \frac{h}{24}(55y'_3 - 59y'_2 + 37y'_1 - 9y'_0)$$

Put these values into formula and get the answers

Q. Describe briefly the Jacobi's method of solving linear equations (02)
ANS: It is an algorithm for determining the solution of a system of linear equation with the largest values in each row and column dominated by the diagonal element.

Q. Find the value of p from the following table of values if $f(.36)$ is to be calculated

x	.2	.3	.4	.5	.6
$f(x)$	.2304	.2788	.3222	.3617	.3979

(03)

Solution:

$$p = \frac{x - x_0}{h}$$

Q. Write the two steps of solving the linear equations using Gaussian Elimination method (03)

Ans: A will be converted into an uppoer triangular form , Above the upper triangular matrix will be reduced by an identity matrix by tow transformations.

Q. Consider the following table of values

x	2	4	6	8	10	12	14
F(x)	23	93	259	569	1071	1813	2843

Use Newton's backwards difference formula to compute f(11.8) . with $x_n = 10$ and p=.9.

ANS:

$$y_x = y_n + p\nabla y_n + \frac{p(p+1)}{2!}\nabla^2 y_n + \frac{p(p+1)(p+2)}{3!}\nabla^3 y_n$$

$$y_x = 2843 + 9(1030) + \frac{9(9+1)}{2!}288 + \frac{9(9+1)(9+2)}{3!}48$$

$$y_{(11.8)} = 19905$$